Title: Darcy's Cove

Brief Overview:

Why does the position(angle) on the beach help Darcy find her grandmother? Where will be the best spots from which to look? Does the arc formed by the cove affect her ability to search for her grandmother? These questions and others will be explored by the students in this unit. The students will collect data and develop formulas to find the angles related to a circle - the central angle, the inscribed angles, and the angle formed by two secants, the angle formed by a tangent and a chord, and the angle formed by two tangents.

Link to Standards:

- **Problem Solving** Students will demonstrate their ability to solve mathematical problems through the use of **Geometer's Sketchpad™** and applying formulas.
- **Communications** Students will predict the outcome of an experiment and explain the reasoning to generate a formula.
- **Reasoning** Students will discover the formula for determining the angle measure or arc measure from chart and manipulating a demonstration sketch on **Geometer's Sketchpad™**.
- **Connections** Students will see a connection between the angle formed in relation to a circle and a situation which requires a search.
- **Algebra** Students will demonstrate their ability to solve an one variable equation.
- **Geometry** Students will manipulate the sketch, make conclusions and apply a formula to similar figures.
- **Measurement** Students will use **Geometer's Sketchpad™** to measure angles and arcs.

Grade/Level:

Grades 9 -12, Geometry

Duration/Length:

This lesson will take 3 or 4 periods (45 min.) or 2 periods (90 minutes).

Prerequisite Knowledge:

Students should have working knowledge of the following:

- How to use **Geometer's Sketchpad™** - selecting points and segments, finding the measure of angles and segments
- Algebraic skills to solve simple one variable equations
- Vocabulary about parts of a circle

Objectives:

Students will :

- work cooperatively in groups.
- write formulas for finding the measure angles related to a circle.
- create a data chart to investigate the problem.
- apply the formulas to new problems.
- explain their reasoning in making conclusions .
- create a new formula for a related situation where one aspect is changed.

Materials/Resources/Printed Materials:

- **Geometer's Sketchpad™**
- Vocabulary List
- Scenario and Predictions Worksheet: Darcy's Cove
- Diagram of Darcy's Cove
- Student Directions
- Data Collection Sheet
- Practice Worksheets 1 - 3
- Script to create Darcy's Cove Figure on Geometer's Sketchpad

Development/Procedures:

Days 1, 2:

1. Divide the class into groups of two and lead the class in a review of vocabulary using the **Vocabulary List**.

2. Present the **Scenario and Predictions Worksheet: Darcy's Cove** and give the students time to make predictions and analyze the problem.

3. Have the students complete the **Data Collection Sheet (Pages 1 and 2 - Data Results)**. The students will create the chart with the results of their findings, write an interpretation of their results, and write a formula for the measure of a central angle and the measure of a inscribed angle.

4. Have the groups report their conclusions about the formulas they have discovered to the class.

5. Have the students complete **Practice Worksheet 1** utilizing their formulas for central angles and inscribed angles.

Days 3, 4:

1. Have the students complete **Practice Worksheet 2 (Pages 1 and 2)** where they create a formula for the angle formed by two secants by investigating the **Geometer's Sketchpad™** document and then utilize the formula to solve similar problems.

2. Discuss how to extend the formulas to tangents and chords (showing the relationship to an inscribed angle) and two tangents (showing the relationship to two secants).

3. Apply this formula to complete **Practice Worksheet 3 (Pages 1 and 2)**.

Evaluation:

- The teacher will observe the work of the groups at the computer and make sure the students are on task.
- Students will complete the Practice Worksheets successfully.

Extension/Follow Up:

- The students could be asked to find the formula to find the measure of an angle formed by two chords intersecting inside a circle.

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Vocabulary List

Name_____
Class _____ Period_____

Define the following in your own words.

circle

semi-circle

radius

chord

diameter

secant

tangent

angle

central angle

inscribed angle

exterior angle

minor arc

major arc

intercepted arc

Scenario and Predictions Worksheet
Darcy's Cove **Name_____**

Class _____ Period_____

Darcy lives near a small circular cove off the Chesapeake Bay. The beach of the cove resembles a semi-circle, with the diameter as the shoreline.

Early one morning Darcy's grandmother launched the boat to run some errands. She was expected to return at noon. Just before noon the sky began to darken in anticipation of bad weather conditions. Darcy became concerned so she decided to meet her grandmother at the cove.

As Darcy follows the path from her house toward the cove she continues to look beyond the radius of the cove in anticipation of her grandmother.

Using mathematical terms, describe what you think Darcy's view will be beyond the radius of the cove for each of the following:

1. As she walks from her home through the field, over the beach to the shoreline.

Darcy decides to use a small dingy to go out onto the cove and continue looking.

2. As she rows the dingy from the shoreline to the edges of the circular cove.

3. As she rows the dingy from the edges of the circular cove to the radius of the cove.

4. As she rows out onto the bay.

As the weather worsens, Darcy considers her other options.

5. walking along the edge of the field from one side to the other.

6. walking along the shore line of the beach from one end to the other.

7. walking along the edge of the cove beyond the beach which is overgrown with vegetation.

ON THE BACK OF THIS PAGE:
Write a paragraph which summarizes your predictions, relates your predictions to what you learned from the demonstration file, and explains any discrepancies.

Student Directions **Name_____**
Class _____ Period_____

1. Complete the **Vocabulary List.**

2. Use the **Diagram of the Cove** to complete the **Scenario and Predictions Worksheet: Darcy's Cove.**

3. Open the demonstration file named **"Darcy's Cove"** from the demo disk.

DO NOT MAKE ANY ADJUSTMENTS TO THE DIAGRAM UNTIL DIRECTED TO DO SO OR YOU MAY NEED TO REINITIALIZE THE DEMONSTRATION DISK.

Verify your predictions from your **Scenario and Predictions Worksheet: Darcy's Cove** using the animation buttons provided.

CAREFULLY !! Write at least one paragraph which summarizes your predictions, relates your predictions to what you learned from the demonstration file, and explains any inconsistencies.

4. Complete the **Data Collection Sheet.**

When you opened the demonstration file named **"Darcy's Cove"** from the disk, the intercepted arc from the left side of the cove to the right side of the cove is less than 90º.

• **To measure the arc** you must first select the points Left Edge of Cove, Point A, Right edge of Cove. Go to the Construct Menu and Drag down to Arc Through Three Points. Go to the measure menu and drag down to arc angle.

BE SURE TO RECORD THE RESULT
on the Data Collection Chart in the column labeled 4!!!

• **To measure the angles** you must first select the points Left Edge of Cove, Darcy's View, Right Edge of Cove. Go to the measure menu and drag down to angle. Move to different spots by dragging Darcy's View.

BE SURE TO RECORD THE RESULTS
on the Data Collection Chart in the column labeled 4!!!

5. Carefully move Left Edge of Cove or Right Edge of Cove or both (look at the arc measure changing as you move the points) until the intercepted arc is equal to 90º.

BE SURE TO RECORD THE RESULTS
on the Data Collection Chart in the column labeled 5!!!

6. Carefully move Left Edge of Cove or Right Edge of Cove or both (look at the arc measure changing as you move the points) until the intercepted arc is greater than 90º.

BE SURE TO RECORD THE RESULTS
on the Data Collection Chart in the column labeled 6!!!

Data Collection Sheet

Name_____
Class _____ Period_____

Measure the arc of the cove for each column of results (from left edge of cove to right edge of cove). Record your results in the table below.

Measure each angle from the left edge of the cove to Darcy's View(the vertex angle) to the right edge of cove for each view listed. Record your results in the table below.

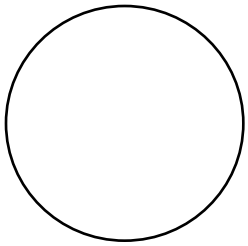
Actual Measure of the intercepted arc / Darcy's View (vertex angle)	4. Arc of Cove < 90 m = _____	5. Arc of Cove = 90 m = _____	6. Arc of Cove > 90 m = _____
A. Darcy's house			
B. Edge of field			
C. Anywhere on the beach*			
D. The shoreline			
E. Before the edge of the cove*			
F. Between the edges of the cove			
G. Beyond the edges of the cove*			

* Measurements may vary.

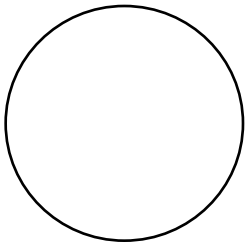
Data Results

Draw a diagram of the angle for Darcy's Locations A, B, D, F, AND G.

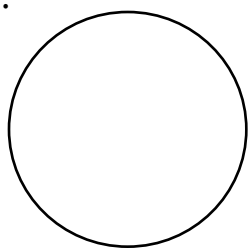
A.



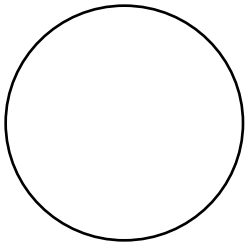
B.



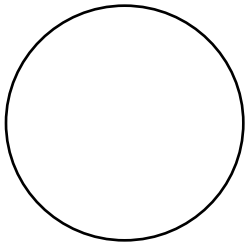
D.



F.



G.



Write at least one paragraph about what you have learned from the data.

Write the two formulas you have discovered in terms of the intercepted arc:

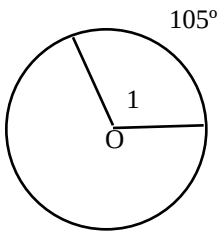
central angle -

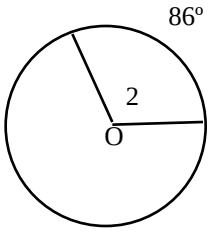
inscribed angle -

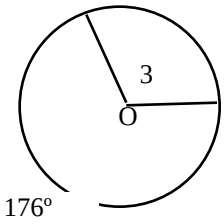
Practice Worksheet 1

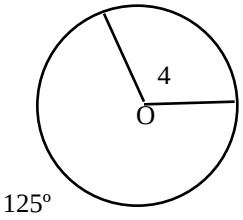
Name_____
Class_____**Period**_____

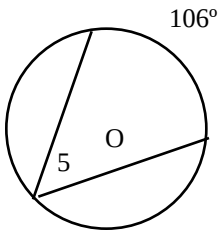
Use the formulas which we discovered to solve the following problems.
O is the center of each circle. Find the measure of the numbered angle.

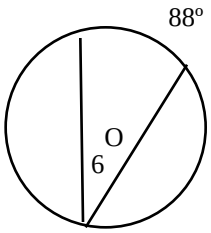


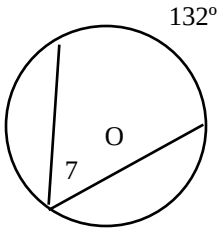


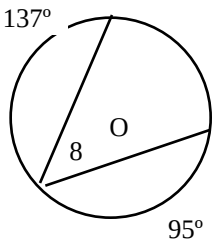


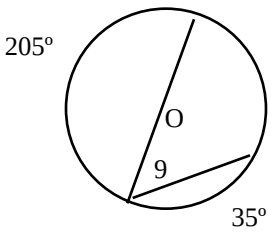


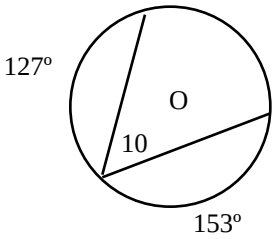




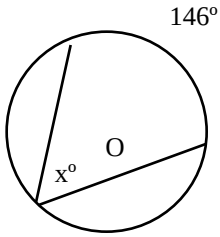


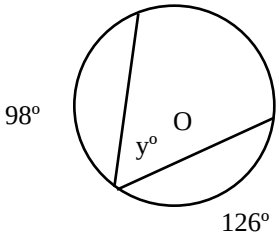


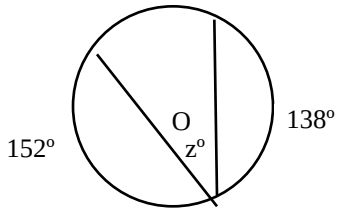




For the following problems find the value of the variable.







Practice Worksheet 2 **Name**_____
Class_____**Period**_____

1. Open **"Darcy's Cove"** and move Darcy's View to a point outside the circle between her house and the beach edge.

2. Select the three points on the arc farthermost away from Darcy's View. Left edge of cove, unnamed point between, right edge of cove - Go to the Construct Menu and Drag Down to Arc through the 3 Points. Go to the Measure Menu and Drag Down to arc angle.

3. Select the ray joining Darcy's View and Left Edge of the Cove and (holding down the shift key) the circle. Go to the Construct Menu and Drag Down to Point of Intersection. Select the ray joining Darcy's View and Right Edge of the Cove and (holding down the shift key) the circle. Go to the Construct Menu and Drag Down to Point of Intersection.

4. Select the three points on the arc closest to from Darcy's View. Go to the Construct Menu and Drag Down to Arc through the 3 Points. Go to the Measure Menu and Drag Down to arc angle.

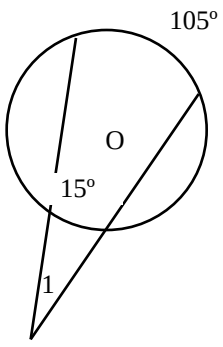
5. Select the points Left Edge of Cove, Darcy's View, Right Edge of Cove. Go to the measure menu and drag down to angle.

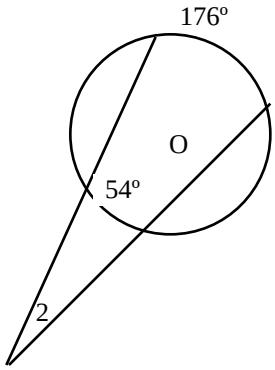
6. Select the measures and go to the Measure Menu and Drag Down to Calculate. Subtract the two arcs(far - close). How does this measure compare to the angle measure?

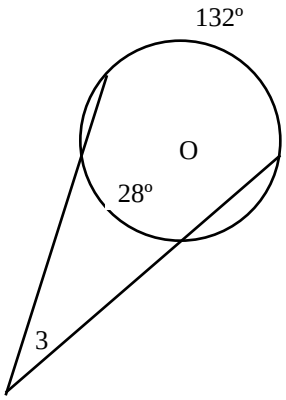
Move Darcy's View around the outside of the circle and look at the measures of the arcs and the angle. How does the difference of the arc measure compare to the angle measure?

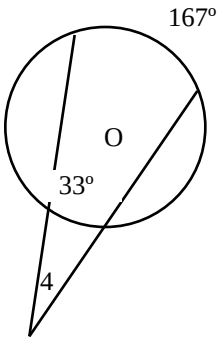
7. Write a formula to show how the arc measures compare to the angle measure.

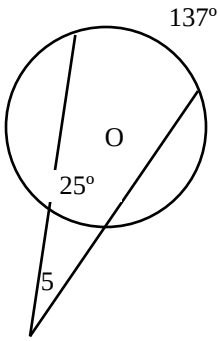
Use the formulas which you have discovered to solve the following problems. O is the center of each circle. Find the measure of the numbered angle.

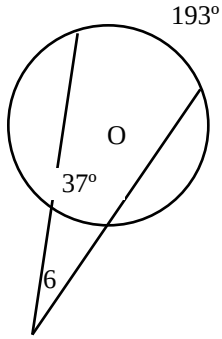




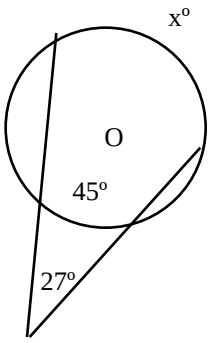


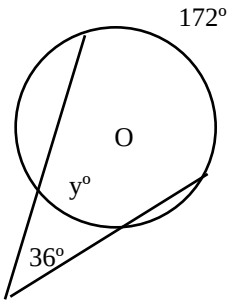


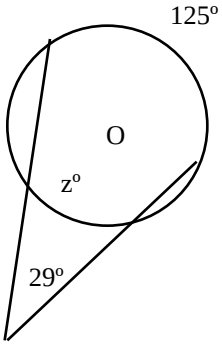




Find the value of the variable for each of the following.

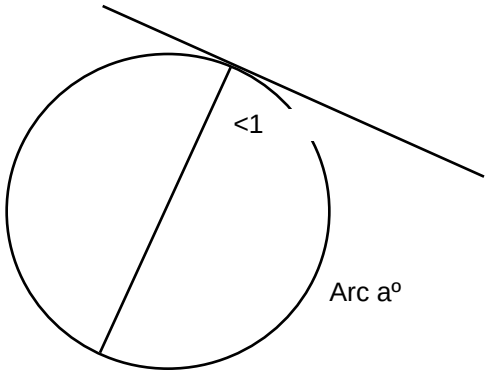




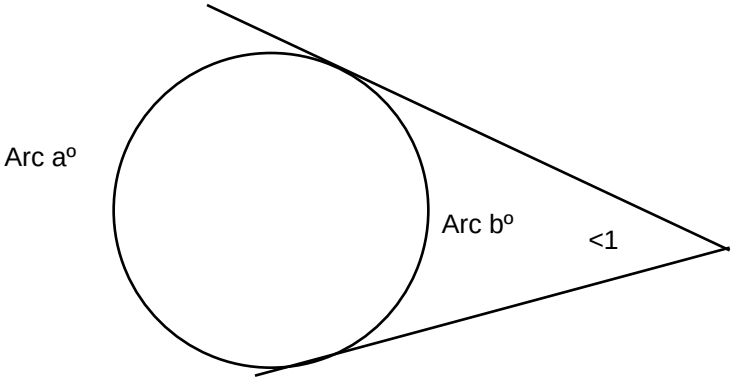


Practice Worksheet 3 **Name**_____
Class_____**Period**_____

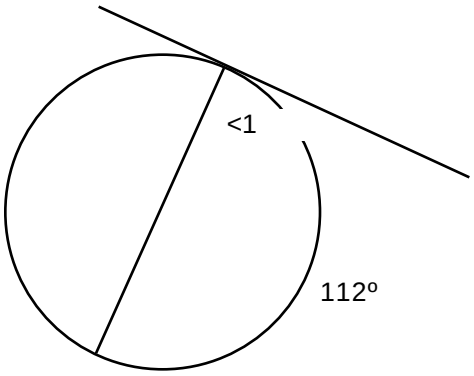
1. Write a formula for the general case of the measure of the angle formed by a chord and a tangent. Use this diagram.

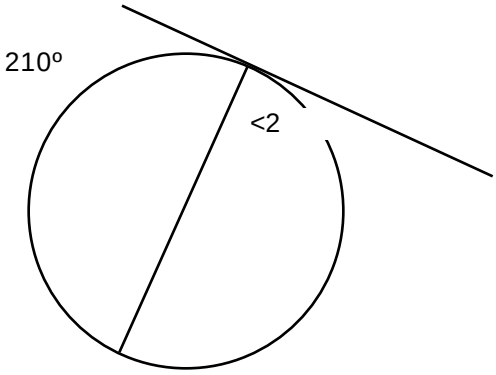


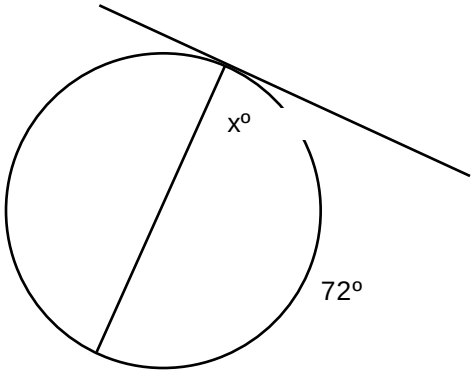
2. Write a formula for the general case of the measure of the angle formed by two tangents. Use this diagram.

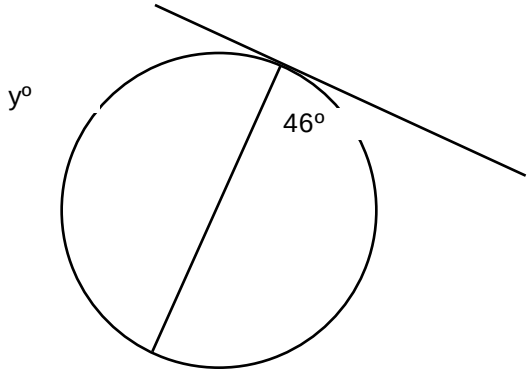


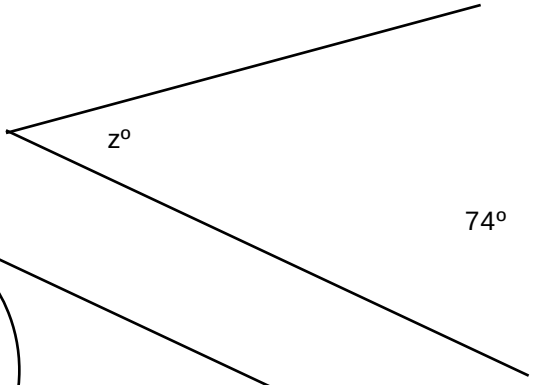
Use the formulas which you have discovered to solve the following problems. O is the center of each circle. Find the measure of the numbered angle.

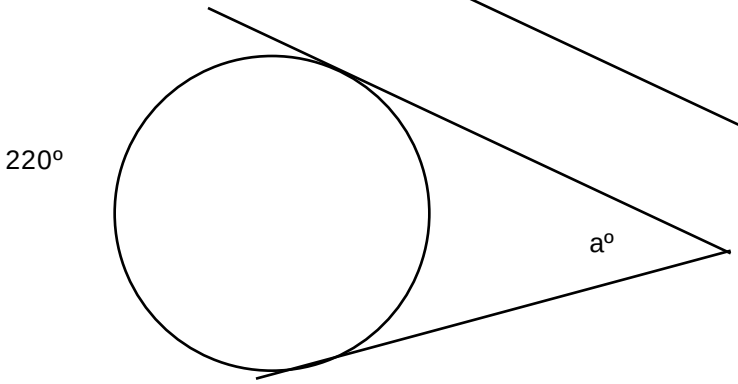


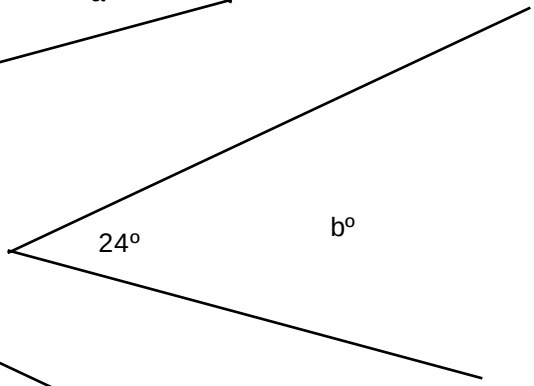



xº
72º


yº
46º


zº
286º
74º


220º
aº


204º
bº
24º

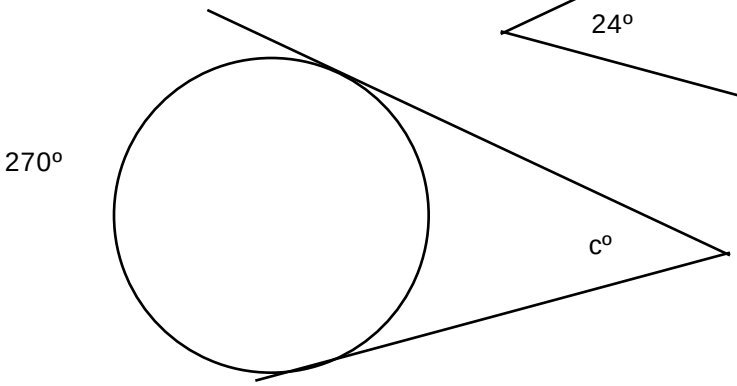
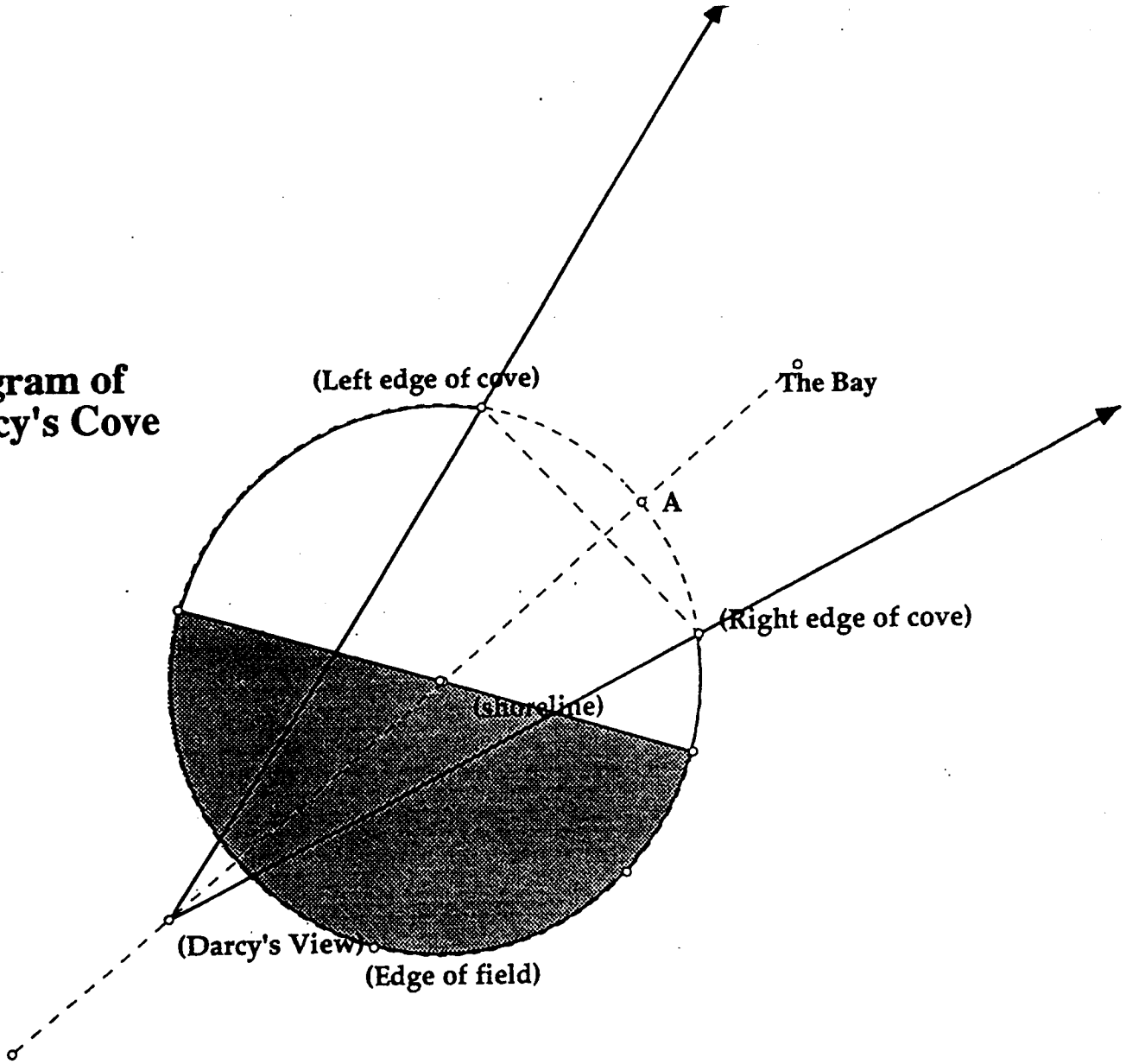

270º
cº

Diagram of Darcy's Cove



- walk from house to shoreline
- shoreline to radius of cove
- past cove
- walk around the edge of field
- walk along shoreline